Math 10B with Professor Stankova
Worksheet, Discussion #25; Tuesday, 4/24/2018
GSI name: Roy Zhao

1 Linear Systems of Differential Equations

1.1 Concepts

1. In order to solve a system of linear differential equations, we represent it in the form $\vec{y}' = A\vec{y}$. Then we find the eigenvalues of A, say λ_1, λ_2. If $\lambda_1 \neq \lambda_2$ are real, then we find the eigenvectors $\vec{v_1}, \vec{v_2}$ and the general solution is of the form $\vec{y} = c_1 e^{\lambda_1 t}\vec{v_1} + c_2 e^{\lambda-2t}\vec{v_2}$.

1.2 Example

2. Find the general solution to the systems of linear differential equations

$$\begin{cases} y_1'(t) = y_1(t) + 4y_2(t) \\ y_2'(t) = 3y_2(t) \end{cases}$$

1.3 Problems

3. True False If 2 is an eigenvalue for A, then 4 is an eigenvalue for A^2.

4. True False If 2 is an eigenvalue of A and 3 is an eigenvalue of B, then $2 \cdot 3 = 6$ is an eigenvalue of AB.

5. True False If two matrices A, B have the same eigenvalues, then they have the same solutions to $\vec{y}' = A\vec{y}$.

6. Find the solution to the systems of linear differential equations

$$\begin{cases} y_1'(t) = 5y_1(t) - 4y_2(t) \\ y_2'(t) = 4y_1(t) - 5y_2(t) \end{cases}$$

with $\vec{y}(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$.

7. Find the general solution to the systems of linear differential equations

$$\begin{cases} y_1'(t) = 2y_1(t) + y_2(t) \\ y_2'(t) = y_1(t) + 2y_2(t) \end{cases}$$

8. Verify that $\vec{x}(t) = \begin{pmatrix} 0 \\ -e^t \\ e^t \end{pmatrix}$, $\vec{y}(t) = \begin{pmatrix} e^{2t} \\ -2e^{2t} \\ 0 \end{pmatrix}$, $\vec{z}(t) = \begin{pmatrix} 0 \\ e^{3t} \\ e^{3t} \end{pmatrix}$ are solutions to $\vec{v}' = A\vec{v}$ where $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$.

9. Under the same notation as the previous problem. Write out the system of linear equations that $\vec{v}' = A\vec{v}$ represents and find the general solution.

10. Still with the same notation, what are the eigenvalues and eigenvectors of A?

2 Miscellaneous

2.1 Problems

11. Let $V = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. Let $A = V \cdot \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix} \cdot V^{-1}$. Calculate A.

12. With the same A as above, calculate the eigenvalues and eigenvectors of A. What do you notice? How does this relate to V?

13. (Challenge) Create a matrix with eigenvalues $\lambda = 1, 2$ and eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ respectively.